The Universal Nature of Zero-Crossing Time and Velocity Scales in Turbulent Shear Flows

Promode Bandyopadhyay*
Systems and Applied Sciences Corporation, Hampton, Virginia
and
A.K.M.F. Hussain†
University of Houston, Houston, Texas

Abstract

RECENT studies have shown that the turbulence production process in shear flows is not entirely random, but rather quasiperiodic in nature. Since the large eddies play a decisive role in these processes in all shear flows, it is instructive to see if they bear any common characteristics. The possible universal distributions[1] of time and velocity scales (T,u_p) in the low-frequency component of the turbulent signals—defined, respectivley, as the interval between successive zero crossings and the intervening absolute peak value of the longitudinal velocity signal—have been explored in a number of flows. These include the self-preserving regions of a turbulent boundary layer, plane jet, circular jet, and plane mixing layer and the initial mixing regions of a plane jet and circular jet. Both T and u_p distributions show universal trends: T distributions agree well with the log normal distribution except in their extreme excursions, whereas u_p distributions are intermediate between log normal and Gaussian distributions.

Contents

The large structure variates studied here are the zero-crossing interval T and the absolute value u_p of the velocity peaks or troughs between two successive zero crossings. The a.c. component in a typical unfiltered u signal of a turbulent shear flow is shown in Fig. 1. Such signals have a slowly varying component on which the high-frequency component rides. This slowly varying part can be obtained by suitably low-pass filtering the signal as shown in the figure. The low-frequency component contains most of the kinetic energy and is considered to be the signature of the large structures. T and u_p are two simple variables describing the time and velocity scales of the signal. To obtain T and u_p, the hot-wire u-signals were digitally filtered and processed.

When $u_p/\bar{u}_p$ is plotted against log $(T/\bar{T})$ following Badri Narayanan,[1] no clear trend becomes apparent (the overbars indicate time mean values). Noting that $(T/\bar{T})^{-1}$ is analogous to frequency, plots of $u_p/\bar{u}_p$ vs $(T/\bar{T})$ for the near-wall point in the boundary layer and those in the plane jet are shown in Fig. 2. Distributions for other flows are similar. The data have not been averaged here in order to retain the scatter in the temporal data. The distributions appear similar to the one-dimensional power spectra of these flowfields. For example, the figure exhibits a decade where the data follow a $-5/3$ trend. The drop is slower at lower values of the ordinate, whereas it is faster at higher values. For the lowest values of the ordinate, the boundary-layer trend is flatter compared to that in the plane jet. Although the exact pattern of the velocity signal has been ignored here in that only the value at the peak or trough has been considered, the familiar power spectral distributions have nevertheless been reproduced. Recall that Badri Narayanan[1] plotted $u_p/\bar{u}_p$ vs log $(T/\bar{T})$ after averaging the data. Although this has apparently led to universal trends, the physical significance of such a plot is not obvious. On the other hand, since the distribution of $u_p/\bar{u}_p$ vs $1/(T/\bar{T})$ is similar to that in a power spectrum, the plots shown in Fig. 2 have a physical significance.

To test whether a population has a log normal or Gaussian distribution, the cumulative probability distribution is commonly plotted on probability paper. The logarithm of the variable falls on a straight line if it has a log normal distribution. The cumulative distribution of log $(T/\bar{T})$ and log $(u_p/\bar{u}_p)$ for all the flowfields is shown in Figs. 3 and 4, respectively. The straight lines shown have been fitted manually by centering at the 50% point. On average, $T/\bar{T}$ is log normal over 10-95%, whereas $u_p/\bar{u}_p$ is so only over 25-65%. The much narrower fit of $u_p/\bar{u}_p$ seems to be related to the fact that $u_p/\bar{u}_p$ varies over 3.5 decades, while $T/\bar{T}$ varies over 2 decades only. See Fig. 2.

The departure of the data from log normality at the ends of the distributions has been computed for both $u_p/\bar{u}_p$ and $T/\bar{T}$. It has been defined as the area that lies between the straight line (log normal) and a mean curve through the data points up to the 1 and 99% points at the two ends. In Figs. 3 and 4, this is indicated by the hatched areas for one case. The estimates show that the departure from log normality is one order of magnitude larger for $u_p/\bar{u}_p$ than for $T/\bar{T}$. Again, this shows that log normality does not describe the distributions of $u_p/\bar{u}_p$ as well as it does those of $T/\bar{T}$.

The mean μ and standard deviation σ of the log normal distributions shown in Figs. 3 and 4 have been compiled. The

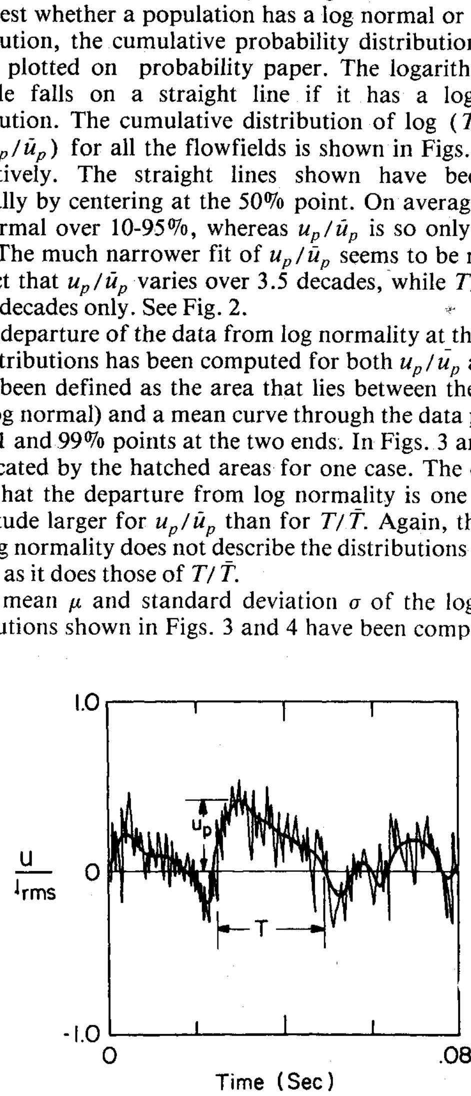

Fig. 1. Definition sketch.

Received March 18, 1983; synoptic received Jan. 8, 1984. Full paper available from National Technical Information Service, Springfield, VA, 22151, at the standard price.

*Principal Member of Technical Staff. Member AIAA.

†Professor. Associate Fellow AIAA.

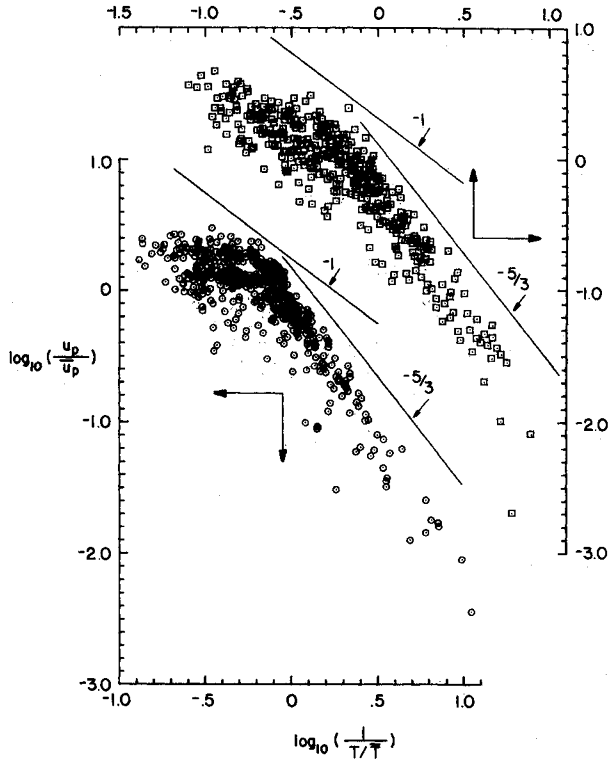

Fig. 2. Relation between u_p and T in a plane jet (upper plot) and a boundary layer (lower plot).

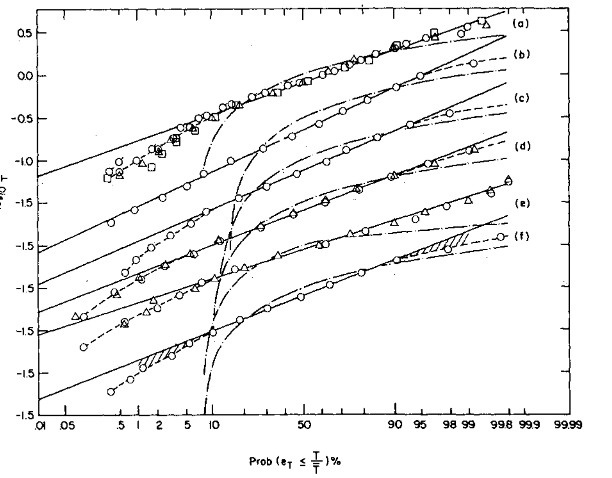

Fig. 3. Cumulative frequency distributions of T in a) boundary layer Y^+ is ○: 8.5, ⦶: 8.5, △: 960, and □: 255); b) plane jet far field (○); c) circular jet (○); d) plane mixing layer (○ and △); e) plane jet mixing layer (○ and ▽); f) axisymmetric mixing layer (○); —— log normal; –·– Gaussian.

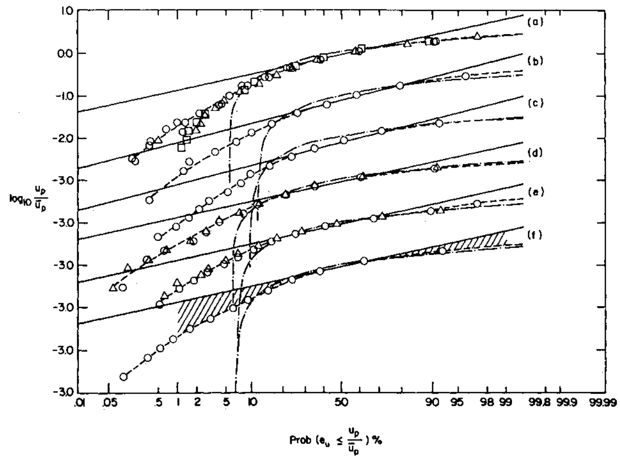

Fig. 4 Cumulative frequency distributions of u_p (symbols same as in Fig. 3).

σ has been calculated as the logarithm of the average of the ratios of $u_p/\bar{u}_p$ or $T/\bar{T}$ at the 50 to 16% point and the 84 to 50% point. The means are close to zero and slightly better so for $u_p/\bar{u}_p$. The σ of both variables is slightly higher in the developed (self-preserving) flowfields of plane and circular jets. This implies a greater dispersion of the eddy sizes and randomness in the jets.

To see how well the data fit a Gaussian trend compared to a log normal distribution, the data should be compared to these two distributions in the same scale. To do this, σ was first computed assuming a Gaussian fit; μ was 1.0 for both the variates. The respective values of σ for $u_p/\bar{u}_p$ and $T/\bar{T}$ were: 0.63 and 0.665 in the boundary layer; 0.671 and 0.691 in the plane jet mixing layer; 0.624 and 0.759 in the plane mixing layers; 0.663 and 0.695 in the axisymmetric mixing layer; 0.841 and 0.919 in the plane jet; and 0.768 and 0.836 in the circular jet. Both the variates have higher σ in the developed flowfields of the plane and circular jets. The Gaussian distributions defined by these μ and σ have been plotted in Figs. 3 and 4.

In Figs. 3 and 4, the distributions are closer to log normal than Gaussian at the lower tail, whereas the Gaussian distribution fits better at the upper tail; around the 50% point, the log normal distribution fits slightly better. We conclude that the $u_p/\bar{u}_p$ distributions are quite universal, but neither log normal nor Gaussian. In Ref. 1, the data processing was manual, the scatter large, and all of the flowfields examined appear to be in developing states. Four of the six flowfields examined here viz. the boundary layer, far fields of the plane and circular jets, and axisymmetric mixing layer are also developing in nature. Note that μ and σ vary much less between these four flowfields. To compare, the average σ of $u_p/\bar{u}_p$ and $\log(T/\bar{T})$ in the present developing flowfields are 0.65 and 0.3, respectively, whereas they were 0.6 and 0.37 in Ref. 1. The Taylor's microscale is $\delta/11$ in our case. Since this is an order of magnitude smaller than δ, we believe that our statistics pertain mainly to large structures.

References

[1] Badri Narayanan, M. A., "Universal Trends Observed in the Maxima of the Longitudinal Velocity Fluctuations and the Zero Crossings in Turbulent Flows," *AIAA Journal*, Vol. 17, May 1979, pp. 527-529.